DMAIC Training

Statistics Primer

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Descriptive vs Inferential Statistics

- **Descriptive Statistics:**
  - Can be defined as those methods involving collection, presentation and characterization of a set of data in order to describe the various features of that data set appropriately
  - Example of descriptive statistics would be the average, median, mode, quartiles etc. of given data set
- **Inferential Statistics:**
  - These are the methods offered by statistics that make possible the estimation of a characteristic of a population or the making of a decision concerning a population based only on sample data

Terminology to familiarize

- **Population**
  - a complete set of data "N"
- **Sample**
  - a subset of data representing the population "n"
- **Mean**
  - the average of the population or sample set
  - Mean of a sample is denoted by X or μ
- **Variance**
  - the (corrected) mean of the squared deviations (It represents the spread of the data)
- **Standard Deviation**
  - is the (positive) square root of the variance
  - Standard Deviation of a sample is denoted by s or σ
- **Median**
  - The mid point
- **Range**
  - The difference between the maximum and the minimum
- **Mode**
  - The most frequently occurring data value
- **Histogram**
  - A frequency distribution for data values

Basic Statistics

- Average/Mean
- Median
- Mode
- Range: Minimum/ Maximum
- Sample Inter-quartile Range
- Sample Variance
- Standard Deviation
- Coefficient of Variation
- Percentile
- Box Plot
- Histogram
- Normal Standard Deviation

What to Describe?

- **What is the "location" or "center" of the Data? ("**measures of location**")**

- **How does the Data vary? ("**measures of variability**")**

Measures of Location

Mean

Median

Mode

Mean

- Another name for average
- If describing a population, denoted as μ, the Greek letter "mu"
- If describing a sample, called "x-bar"
- Appropriate for describing measurement data
- Seriously affected by unusual values called "outliers"

Calculating Sample Mean

Formula:

$$\overline{X} = \frac{\sum X_i}{n}$$

That is, add up all of the data points and divide by the number of data points

Data (of process defects): **2 8 3 4 1**

Sample Mean = (2 + 8 + 3 + 4 + 1) / 5 = 3.6

Do not round…Mean need not be a whole number

Median

- Another name for 50th percentile
- Appropriate for describing measurement Data
- "Robust to outliers", that is, not affected much by unusual values

Calculating Sample Median

- **Order data from smallest to largest:**

If **odd** number of data points, the median is the middle value

Data (# process defects): **2 8 3 4 1**

Ordered Data: **1 2 3 4 8**

Median

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Calculating Sample Median

Order data from smallest to largest

If **even** number of data points, the median is the average of the two middle values

Data (# of process defects): **2 8 3 4 1 8**

Ordered Data: **1 2 3 4 8 8**

Median = (3 + 4) /2 = 3.5

Mode

- The value that occurs most frequently
- One data set can have many modes
- Appropriate for all types of data, but most useful for categorical data or discrete data with only a few number of possible values
- 2,3,3,4,5,2,3,4,5,3,4,3,3,4,5,2,
- 2,2,2,**3,3,3,3,3,3**,4,4,4,4,5,5,5

Measures of Variability

- Range
- Interquartile Range
- Variance and Standard Deviation
- Coefficient of Variation

All of these measures are appropriate for measurement data only

Range

- The difference between largest and smallest data point
- Highly affected by outliers
- Best for symmetric data with no outliers

R = largest obs. - smallest obs.

or, equivalently

$R = x_{max} - x_{min}$

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Interquartile Range

- The difference between the "third quartile" (75th percentile) and the "first quartile" (25th percentile). So, the "middle-half" of the values
- IQR = Q3-Q1
- Robust to outliers or extreme observations
- Works well for skewed data

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What is Standard Deviation?

- It is the typical (standard) difference (deviation) of an observation from the mean

- Think of it as the average distance a data point is from the mean.

NOTE : Standard deviation is an important measure from Six Sigma perspective and will be used in many examples as well as calculations.

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

xi - is individual data
x – is mean value of the sample
n – is the number of samples

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Sample Standard Deviation

- **Formula for Std Deviation**

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

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Measures of Variation - Some Comments

- Range is the simplest, but is very sensitive to outliers
- Variance units are the square of the original units
- Interquartile range is mainly used with skewed data (or data with outliers)
- We will use the standard deviation as a measure of variation often in this course

How to Calculate Percentile

Percentile value = Total number of readings * (decided percentile) + (decided percentile)

For example, the median is the 50th percentile

We want to determine the median of a dataset of 40 nos.

0.5 * 40 + 0.5 = 20.5

Thus the median will be the reading between the 20^{th} and 21^{st} reading

The third quartile will be 0.75 * 40 + 0.75 = 30.75

That is the extrapolated reading between the 30^{th} and the 31^{st} reading

Boxplot - a 5 Number Summary

- Smallest Observation (Min)
- Q_1
- Q_2 (median)
- Q_3
- Largest Observation (Max)

Boxplot Example

- Smallest observation = 3.20
- Q_1 = 43.645
- Q_2 (median) = 60.345
- Q_3 = 84.96
- Largest observation = 124.27

Creating a Boxplot

- Create a scale covering the smallest to largest values
- Mark the location of the five numbers
- Draw a rectangle beginning at Q1 and ending at Q3
- Draw a line in the box representing Q2, the median
- Draw lines from the ends of the box to the smallest and largest values
- Some software packages that create boxplots include an algorithm to detect outliers. They will plot points considered to be outliers individually

Boxplot Example

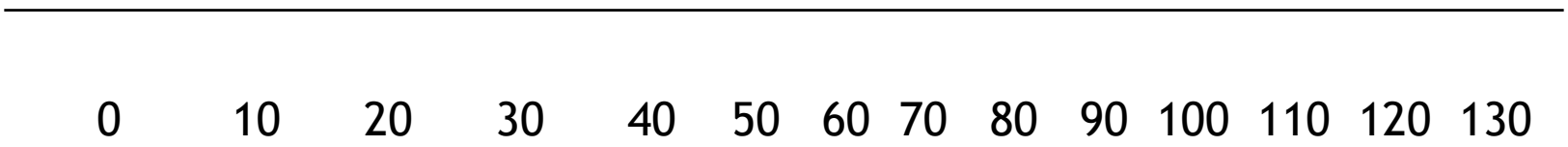
Min = 3.20

Q_2 = 43.35

Max = 124.27

Q_1 = 33.645

Q_3 = 84.96



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Boxplot Interpretation

- The box represents the middle 50% of the data, i.e., IQR = length of box
- The difference of the ends of the whiskers is the range (if there are no outliers)
- Outliers are marked by an * by most software packages
- Boxplots are useful for comparing two or more samples
  - Compare center (median line)
  - Compare variation (length of box or whiskers)

Histograms

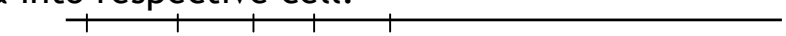
- Building Histograms

More Than	Up to	Cell Size
0	1	1
1	2	2
2	4	3
4	8	4
8	16	5
16	32	6
32	64	7
64	128	8
128	256	9
256	512	10

Based on number of data points, select # cells from table

- Determine cell interval.
- Building the limit histogram axis.
- Building histogram by placing data into respective cell.

$$\frac{\text{Max - Min}}{\text{\# Cells}}$$



Lowest Limit = Min - (0.5) x Cell Size

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Histograms (cont)

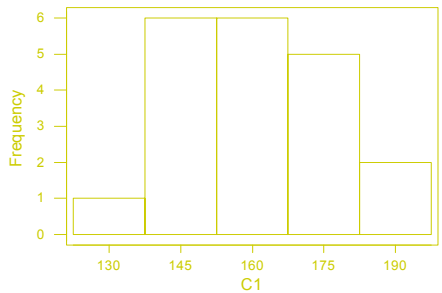
- **Guideline for Forming the Cell Intervals:**
  - Use intervals of equal length with midpoints at convenient numbers
  - For small data set, use a small number of intervals
  - For a large data set, use more intervals
  - Make sure that a data point can only be located in one cell
- **Histograms:**
  - What: Graph of frequency of occurrence of an event
  - Why: Used to breakdown the distribution by number of occurrences per event to further define the data
  - How: For each event, or group of data, plot number of times the event occurs. If needed, add cumulative percent. From left to right add sequentially each event percentage

Histogram: Example

- A Histogram is a bar graph that shows the number (relative frequency) of each data point within a cell or interval. Each bar covers the interval and is centered at the mid point. Using minitab and the following data, create a histogram.

Sample	Value
1	140
2	145
3	160
4	140
5	155
6	165
7	150
8	190
9	138
10	155

Sample	Value
11	153
12	145
13	170
14	175
15	175
16	180
17	135
18	157
19	170
20	185



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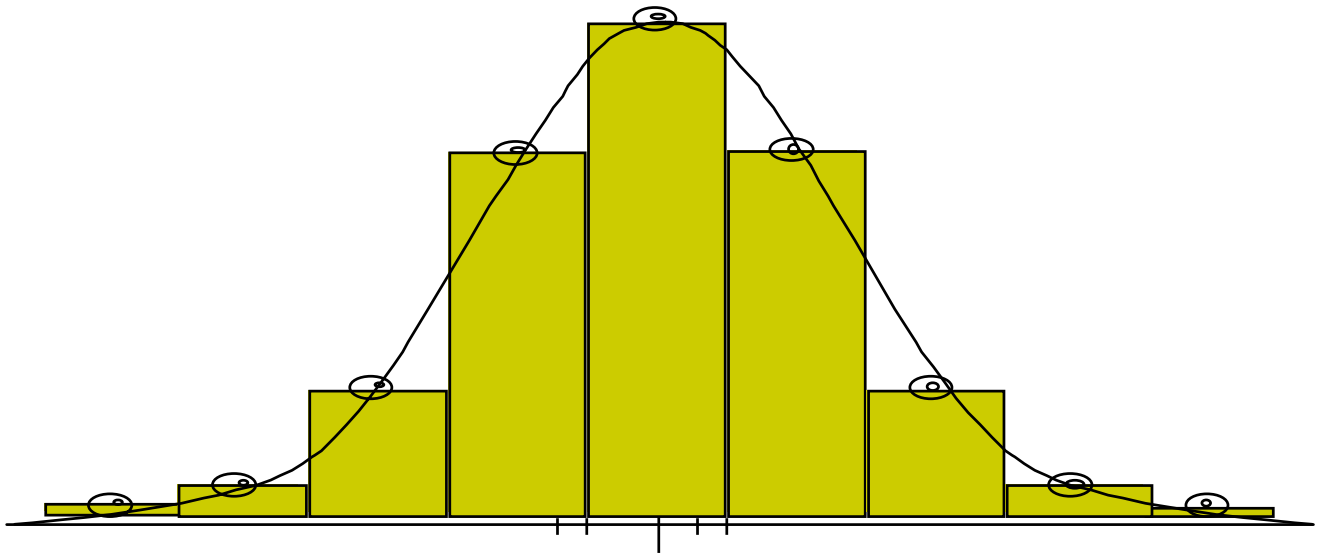
Normal Curve

Smooth Line Connects the Center of Every Bar of the Histogram



The Characteristic of a normal curve is that Mean, median and the mode are the same.

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Probability Distribution of Continuous Random Variable:

- **Properties of a Normal distribution:**
  - It is a bell shaped curve that is symmetrical
  - Its measure of central tendency (mean, median, mode) are all identical
  - The probability that a point is x distance below the average is the same as that of a point to be x distance greater than the average
  - Measures that we expect to find normally distributed would be absenteeism, abandon rate, service level

Features of a Normal Standard Distribution

- 100% of the area under the normal curve lies between ± infinity, we may calculate that area which lies beyond the specification limit. Both above the USL (Upper Specification Limit) or / and below the Lower Specification Limit (LSL) Doing so would reveal the random chance probability of creating a defect

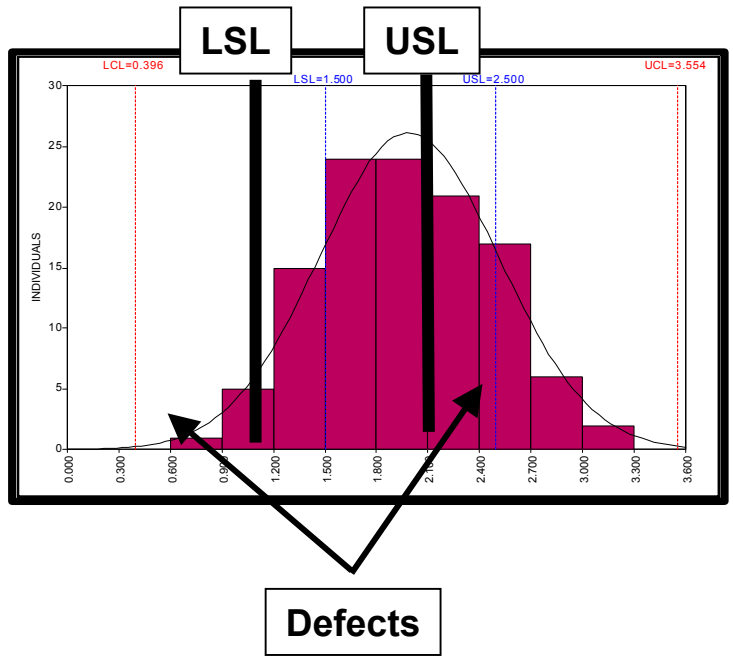
In order to use the normal table to

Accommodate any normal distribution, we need only to standardize our data:

$Z = (X - \mu) / \sigma$

The Z score, denoted by "Z", corresponds to the value at X, and is the

distance between X and the mean (μ) in units of the standard deviation



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